Comment on "Variable viscosity and slip velocity effects on the flow and heat transfer of a power-law fluid over a non-linearly stretching surface with heat flux and thermal radiation " Ahmed M. Megahed , Rheologica Acta, 51, pp. 841847 (2012) .

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Abstract: In the above paper [1], the author claimed that the similarity solution exists only when the exponent m=$1/3$ [1]. It is shown that this condition insufficient to support his claim. It is found that the similarity solution exists in two cases only :

(i) m=$1/3$ and the slip parameter $\lambda = 0$ (no-slip surface condition);

(ii) n=1(Newtonian fluid) and m=1 (linear stretching surface). Also, it is shown that the local skin –friction coefficient defined by the author in ref. [1] is wrong. Consequently, the viscosity parameter α, the radiation parameter R and the Prandtl number P_r have opposite effects on the local skin –friction coefficient than that obtained in ref. [1] .

Introduction

The problem of boundary – layer flow and heat transfer of a non- Newtonian power-law fluid past a non-linear stretching surface with variable heat flux in the presence of thermal radiation and wall slip velocity investigated numerically in ref. [1]. The author pointed out that the similarity solution exists only when the exponent $m = 1/3$. Also , the author found that α and R have the effect of enhancing the local skin-friction coefficient ($\left| f''(0) \right|^n$) while P_r has an opposite effect.

The aim of this comments is to show that the condition $m = 1/3$ insufficient to obtain a similarity solution as well as the definition of the local skin-friction coefficient is wrong .

Comments :

1. In ref.[1] , the non-Newtonian Prandtl number is defined as :

$$P_r = \frac{\rho c_p}{\kappa}\left(c^{3(n-1)} x^{(3m-1)(n-1)} \left(\frac{\mu_\infty}{\rho}\right)^2 \right)^{\frac{1}{n+1}} , \qquad (1)$$

and the slip velocity parameter is :

$$\lambda = \frac{\lambda_1 U \rho}{\mu_\infty} R_{e_x}^{\frac{-1}{n+1}} \text{ (the correct definition of } \lambda \text{ is } \lambda = \frac{\lambda_1 \rho U\, e^{-\alpha\theta(0)}}{\mu_\infty} Re_x^{\frac{-1}{n+1}}), \qquad (2)$$

where λ_1 is the slip coefficient having dimension of length , $U = cx^m$ ($c > o$ is a constant) is the non-linear stretching velocity and $R_{e_x} = \frac{\rho U^{2-n} x^n}{\mu_\infty}$ is the local non-Newtonian Reynolds number , then Eq.(2) becomes :

$$\lambda = \frac{\lambda_1^* c^{\frac{2n-1}{n+1}} \rho^{\frac{n}{n+1}}}{\mu_\infty^{\frac{n}{n+1}}} x^{\frac{(2m-1)\ n-m}{n+1}} . \qquad (3)$$

where $\lambda_1^* = \lambda_1 e^{-\alpha\theta(0)}$.

The similarity solution exists if, the Prandtl number Pr in Eq. (1) and the slip parameter given by Eq. (3) are independent on x.

The author in ref.[1] claimed that a similarity solution exists only when $m = 1/3$. If this is correct, Eq.(3) gives unrealistic value $n = -1$ which in contradict with the rheological index $n > 0$. Also, all results obtained in ref.[1] with $m = 1/3$ and various values of n ($0 < n < 2$) are not belongs to a similarity solution. From Eq.(1) and (3), it is clear that a similarity solution exists in two cases: (i) $m = 1/3$ and $\lambda = 0$ (no-slip at the surface);
(ii) $m = 1$ (linear stretching surface) and $n = 1$ (Newtonian fluid).

2. The local skin-friction coefficient in ref.[1] defined as

$$C_{f_x} = -2 Re_x^{\frac{-1}{n+1}} \left| f''(0) \right|^{n-1} f''(0) . \qquad (4)$$

The correct local skin-friction coefficient is:

$$C_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{-2(\mu\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y})_{y=0}}{\rho U^2} = -2\,Re_x^{\frac{-1}{n+1}}\left|f''(0)\right|^{n-1}f''(0)e^{-\alpha\theta(0)}. \quad (5)$$

This is common error in many papers [2- 6] .

Using the correct form of C_{f_x} and the results obtained in Table 2 for $\left|f''(0)\right|^n$ (see ref.[1]), it is shown that , in contrary to the results obtained in ref.[1] , the Prandtl number have the effect of enhancing the local skin-friction coefficient while both the viscosity parameter α and the radiation parameter R have reducing it .

Conclusion

The similarity solution exists only when $m = 1/3$ and $\lambda = 0$. Also, the local skin-friction coefficient should be $C_{f_x} = 2\,Re_x^{\frac{-1}{n+1}}\left|f''(0)\right|^n e^{-\alpha\theta(0)}$.

References

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